

# Will Solar Cycles 25 and 26 Be Weaker than Cycle 24 ?

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**Abstract** The study of variations in solar activity is important for understanding the underlying mechanism of solar activity and for predicting the level of activity in view of the activity impact on space weather and global climate. Here we have used the amplitudes (the peak values of the 13-month smoothed international sunspot number) of Solar Cycles 1–24 to predict the relative amplitudes of the solar cycles during the rising phase of the upcoming Gleissberg cycle. We fitted a cosine function to the amplitudes and times of the solar cycles after subtracting a linear fit of the amplitudes. The best cosine fit shows overall properties (periods, maxima, minima, *etc.*) of Gleissberg cycles, but with large uncertainties. We obtain a pattern of the rising phase of the upcoming Gleissberg cycle, but there is considerable ambiguity. Using the epochs of violations of the Gnevyshev-Ohl rule (G-O rule) and the ‘tentative inverse G-O rule’ of solar cycles during the period 1610–2015, and also using the epochs where the orbital angular momentum of the Sun is steeply decreased during the period 1600–2099, we infer that Solar Cycle 25 will be weaker than Cycle 24. Cycles 25 and 26 will have almost same strength, and their epochs are at the minimum between the current and upcoming Gleissberg cycles. In addition, Cycle 27 is expected to be stronger than Cycle 26 and weaker than Cycle 28, and Cycle 29 is expected to be stronger than both Cycles 28 and 30. The maximum of Cycle 29 is expected to represent the next Gleissberg maximum. Our analysis also suggests a much lower value (30–40) for the maximum amplitude of the upcoming Cycle 25.

## 1. Introduction

Solar activity varies on many timescales, from a few minutes to a few decades. Solar activity affects us in many ways. It causes problems in both the space- and the ground-based technologies, impacts on Earth’s magnetosphere and ionosphere, and also may have an influence on terrestrial climate. Therefore, predicting solar activity on all timescales is important (Svalgaard, 2013; Hathaway,

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2015). A number of attempts have been made to predict the amplitude of a sunspot cycle by using several properties of previous cycles (see Kane, 2007; Pesnell, 2008, 2012; Obridko and Shelting, 2008; Hathaway, 2009). The amplitude of a solar cycle near the time of solar cycle minimum can be predicted using precursors such as geomagnetic activity and polar magnetic fields. This method seems to be related to the mechanism of solar cycle, *i.e.* it is believed that the polar fields act as a “seed” for the dynamo producing the next cycle (Jiang, Chatterjee, and Choudhuri, 2007 and references therein). From this method it seems possible to predict the amplitude of a cycle by about four years in advance with reasonable accuracy. Recently, Cameron, Jiang, and Schüssler (2016) and Hathaway and Upton (2016) used simulations of the polar field at the minimum of the next Cycle 25 (at the start of 2020) to predict that the amplitude of Cycle 25 will be similar to that of the current Cycle 24.

Dikpati and Gilman (2006) made an unsuccessful prediction of the amplitude of Cycle 24 on the basis of a flux-transport dynamo model. While the solar meridional flow speed varies (Basu and Antia, 2003; Javaraiah and Ulrich, 2006; Javaraiah, 2010, Komm *et al.*, 2015 and references therein), in this model a constant value was assumed. The polar field measurements required as input to the predictions based on the flux-transport dynamo models are available only for about three cycles. The predictions based on the polar fields at cycle minimum could be uncertain because it was unclear exactly when these polar field measurements should be taken. Predictions based on the polar fields for previous cycles have given different values at different times (Hathaway, 2015). Sunspot Cycle 24 has two peaks. The second peak is larger than the first peak (this is unusual) and the gap (about two years) between them is also slightly larger than normal. The reason could be that Cycle 24 proceeds at different speeds in the northern and southern hemispheres. The southern hemisphere sunspot peak of Cycle 24 occurred in April 2014 and was about 30% stronger than the northern hemisphere peak, which occurred in September 2011 (Hathaway, 2015; Javaraiah, 2016; Deng *et al.*, 2016, and references therein), but the polar fields had a similar strength in both the northern and southern hemispheres at the end of Cycle 23. Therefore, the polar fields may be going through different dynamo processes in the northern and southern hemispheres, which produces two different seeds. That is, dynamo ingredients seem to work differently in the northern and southern hemispheres, creating asymmetries between these hemispheres (see *e.g.* Belucz and Dikpati, 2013; Shetye, Tripathi, and Dikpati, 2015). Overall, polar fields do not seem to be a simple precursor.

Gnevyshev and Ohl (1948) investigated the variability of the sunspot index using the Zürich database (1755–1944) and found that the sum of the annual international sunspot number ( $R_Z$ ) during an odd-numbered cycle exceeds that of the preceding even-numbered cycle, now commonly referred to as the Gnevyshev-Ohl rule (G-O rule). They also showed that the sum of annual  $R_Z$  of an even-numbered cycle correlated well (sample size  $N = 8$ , the correlation coefficient  $r = 0.91$  is significant on a 99.8% confidence level) with that of the following odd-numbered cycle and that the correlation was weak ( $r = 0.5$  is significant on 79.3% confidence level) with that of the preceding odd-numbered cycle (also see Sýkora and Storini, 1997). It has been believed that it is possible

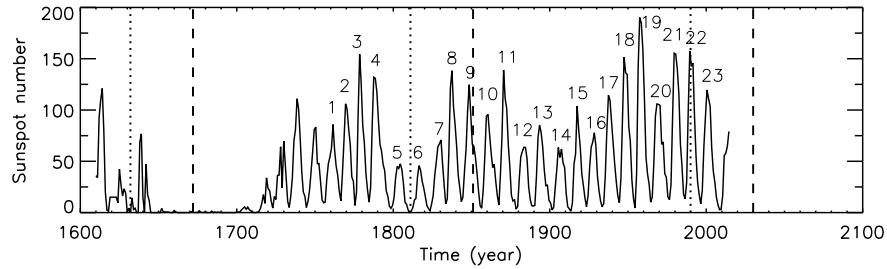
to predict the amplitude of an odd-numbered cycle using the G-O rule. However, the G-O rule is occasionally violated. Predictions of the amplitude of Cycle 23 based on this rule failed drastically. Recently, Javaraiah (2007, 2008, 2015) used the north-south asymmetry property of the sunspot activity (the sum of the areas of sunspot groups in low latitudes) to show that it is possible to predict the amplitude of a cycle (valid for both odd- and even-numbered cycles) approximately nine years in advance with reasonable accuracy ( $N = 11$ ,  $r = 0.966$  is significant on a 99.99% confidence level). Javaraiah (2015) predicted that the amplitude of the upcoming Cycle 25 will be considerably lower than that of the current Cycle 24, *i.e.* a violation of the G-O rule by the Cycle pair (24, 25). Tlatov (2015) analyzed the numbers of sunspot groups since 1610 and found that the G-O rule displays cycles of inversion with a period of 200 years; the latest inversion occurred in Cycle pair (22, 23), and several upcoming odd-numbered cycles should be weaker than their respective preceding even-numbered cycles.

Gleissberg (1939) found a periodicity of seven or eight cycles in cycle amplitudes from 1750 to 1928. This property of the solar activity, known as the Gleissberg cycle, is now well established (Zolotova and Ponyavin, 2014; Vázquez *et al.*, 2016; Komitov *et al.*, 2016). Some authors have suggested that the activity is currently at the minimum of the recent Gleissberg cycle (Javaraiah, Bertello, and Ulrich, 2005a; Zolotova and Ponyavin, 2014; Gao, 2016). In this article we attempt to predict the relative amplitudes of the solar cycles that correspond to the rising phase of the upcoming Gleissberg cycle. For this, we have analyzed the yearly and the cycle-to-cycle variations in  $R_Z$ . Essentially, we use the aforesaid prediction by Javaraiah (2015) and long-term trends in the yearly and cycle-to-cycle variations in  $R_Z$ . We also use the epochs of the steep decrease in the Sun's orbital angular momentum about the solar system barycenter, which seem to help inform epochs of violations of G-O rule (Javaraiah, 2005). That is, we use these epochs to identify some long-term trends in  $R_Z$  for the present purpose. Our aim here is not for to establish the solar activity connection to the solar system dynamics (which is beyond the scope of the present analysis).

In the next section we describe the data analysis and results. In Section 3 we summarize the conclusions and present a brief discussion.

## 2. Data analysis and results

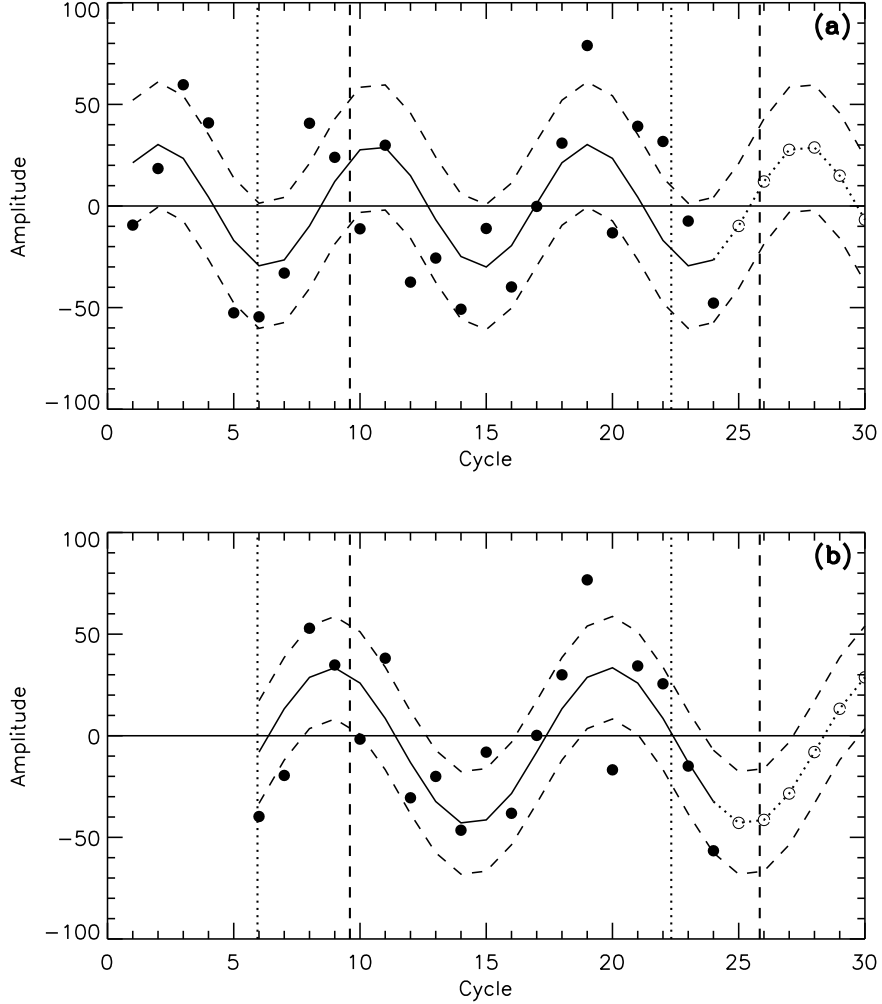
Figure 1 shows the variation in the yearly mean  $R_Z$  during the period 1610–2015, which is taken from <http://www.ngdc.noaa.gov/>. The same figure also shows the epochs when the orbital angular momentum of the Sun about the solar system barycenter decreased steeply (about 25% lower than mean drop; also see Javaraiah, 2005) during the period 1600–2099. Close to these epochs the orbital motion of the Sun is retrograde, *i.e.* the rate of change in orbital angular momentum of the Sun is slightly negative. However, at these epochs the orbital positions of the giant planets are different, hence there are slight differences in the corresponding values of the Sun's distance from the solar system barycenter, orbital velocity, orbital angular momentum and its rate of change (see Table 1 in Javaraiah, 2005). In 1632, 1811, and 1990 (indicated with



**Figure 1.** Variations in the yearly mean  $R_Z$  (before 1950, the data represent the observations that were compiled by Hoyt and Schatten (1998) from numerous sources). Near the peak of each solar cycle the corresponding Waldmeier cycle number is given. The *dotted vertical lines* (in 1632, 1811, and 1990) and *dashed vertical lines* (in 1672, 1851, and 2030) are drawn at the epochs of steep decrease in the Sun’s orbital angular momentum about the solar system barycenter. Close to all these epochs, the orbital motion of the Sun is retrograde (the rate of change in orbital angular momentum of the Sun is slightly negative). At the epochs that are indicated with *dotted vertical lines*, the values of the Sun’s distance from the solar system barycenter, orbital velocity, and orbital angular momentum are slightly higher than those at the epochs that are indicated with *dashed vertical lines* (see Table 1 in Javaraiah, 2005).

dotted vertical lines), the values of the Sun’s distance from the solar system barycenter, orbital velocity, and orbital angular momentum are slightly higher than those in 1672, 1851, and 2030 (indicated with dashed vertical lines). The orbital angular momentum data were provided by Ferenc Váradi. He had derived these data using the Jet Propulsion Laboratory (JPL) DE405 ephemeris for the period 1600–2099. The positions and velocities required may be obtained from JPL’s online Horizons ephemeris system (see Seidelmann, 1992; Giorgini *et al.*, 1996; Standish, 1998). We have fitted a cosine function to the amplitudes of the solar cycles (the peak values of 13-month smoothed  $R_Z$ , which are also taken from <http://www.ngdc.noaa.gov/>) after removing the long-term trend by subtracting a linear fit to the amplitude values as a function of time, in a similar manner as was done for the strengths of Cycles 6–23 by Javaraiah, Bertello, and Ulrich (2005a). We determined the best-fit cosine function for all Cycles 1–24 and only for Cycles 6–24. The results are shown in Figures 2(a) and 2(b). In these figures the extrapolated values for Cycles 25–30 are also shown. The fitting of the Cycles 6–24 data (*root-mean-square (rms) deviation*  $\sigma = 25.2$  and  $\chi^2 = 237$ ) is slightly better than that of all Cycles 1–24 data ( $\sigma = 30.8$  and  $\chi^2 = 433$ ). The values of  $\chi^2$  are very high (the values  $\chi^2$  significant at 17 and 23 degrees of freedom are 27.587 and 35.172, respectively, at the 5% level of significance, *i.e.* for  $P = 0.05$ ). However, the difference between the  $\chi^2$  values are not critical for the current study (see below).

The continuous curves in both Figures 2(a) and 2(b) represent the Gleissberg cycles, but have a substantial difference in their periods: the former shows the cycles of period 8.5 solar cycles, whereas the latter shows the cycles of period 11.0 solar cycles. Several authors have found changes in the period of Gleissberg cycle (Garcia and Mouradin, 1998; Hathaway, Wilson, and Reichmann, 1999; Rozelot, 1994; Ogurtsov *et al.*, 2002). Hathaway (2015) examined the amplitudes of Cycles 1–23 and found a 9.1-cycle periodicity. In both Figures 2(a) and 2(b) the

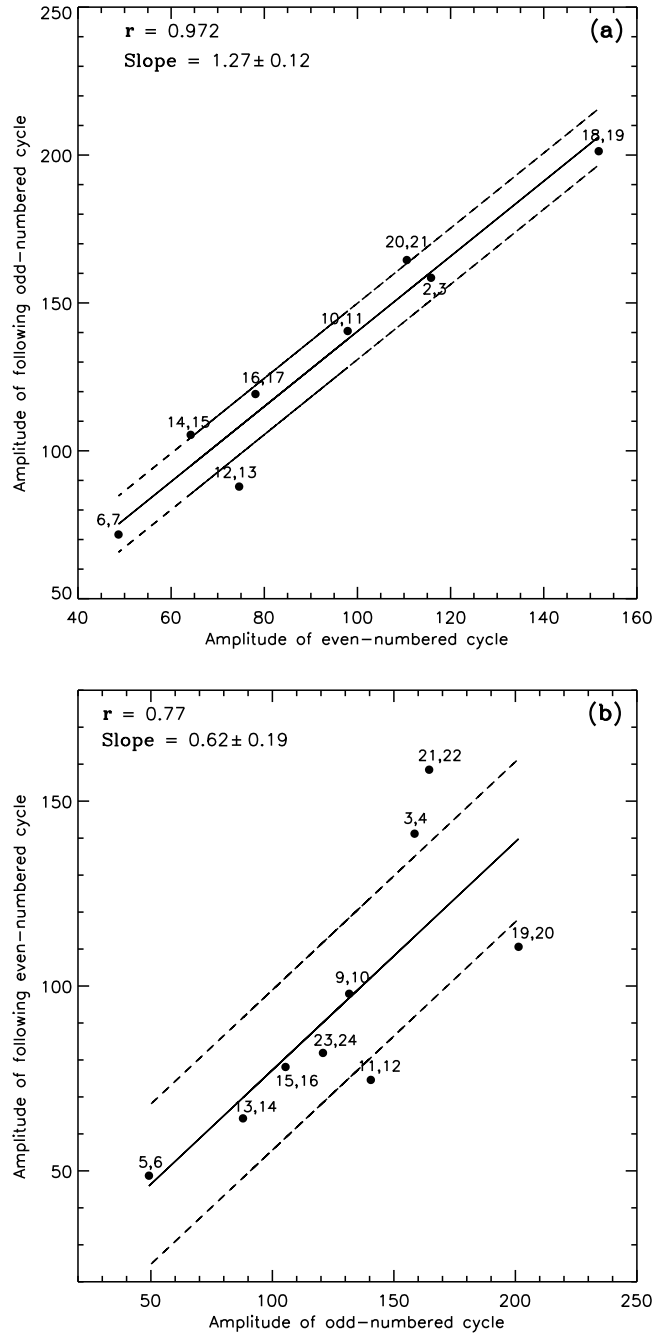


**Figure 2.** Plot of the amplitudes (*filled circles*), *i.e.* the peak values of the 13-month smoothed  $R_Z$ , of the sunspot cycles after subtracting a linear fit from the cycle amplitudes as a function of time. The *continuous curve* represents the best cosine fit. The *open circles* connected by the *dotted curve* represent the extrapolated amplitudes of the upcoming Cycles 25–30. The *dashed curve* represents the *rms deviation* ( $\sigma$ ). Same as Figure 1: the *dotted* and *dashed vertical lines* are drawn at the epochs of steep decrease in the Sun's orbital angular momentum. (a) The entire cycle data set (1–24), the corresponding  $\sigma = 30.8$  and  $\chi^2 = 433$ , and (b) Cycles 6–24 data, the corresponding  $\sigma = 25.2$  and  $\chi^2 = 237$ .

minima of the Gleissberg cycles are reasonably well defined (lower uncertainty, i.e. the data points are within the dashed curve drawn at  $1\sigma$  level). However, Figure 2(b) clearly shows that Cycles 25 and 26 are at the minimum between the current and the upcoming Gleissberg cycles, whereas Figure 2(a) shows that the corresponding minimum is at Cycle 24. That is, there is a considerable difference between the extrapolated portions of the curves in Figures 2(a) and 2(b), which show different epochs for the minimum and the maximum of the upcoming Gleissberg cycle. Overall the uncertainties in the cosine fittings are large ( $\chi^2$  values are much higher than the corresponding values at the 5% level of significance). Hence, it is not possible to infer the amplitudes of the upcoming cycles or even their relative values from either of the curves shown in Figures 2(a) and 2(b). In each of these figures the extrapolated values have large uncertainties. From the trends in  $R_Z$  shown in Figure 1 may be possible to infer the relative values of the amplitudes of the upcoming solar cycles as argued below.

As can be seen in Figure 1, the Cycle pairs (4, 5), (8, 9), and (22, 23) violated the G-O rule. Excluding these cycle pairs we find  $r = 0.972$  (significant on the 99.99% confidence level), the slope is 10.6 times larger than the standard deviation, and  $\chi^2 = 6.43$  (insignificant on 5% level) of the best-linear fit between amplitudes of the remaining pairs ( $N = 8$ ) of even- and the following odd-numbered cycles (see Figure 3(a)). Although, as mentioned in Section 1, the correlation between the sizes of an even-numbered cycle and preceding odd-numbered cycle is weak, Figure 1 shows that the amplitude of an even-numbered cycle is frequently smaller than its preceding odd-numbered cycle. (In the case of Cycle pairs (5, 6) and (21, 22) the amplitudes of the odd-numbered Cycles 5 and 21 are negligibly larger than those of the even-numbered Cycles 6 and 22) That is, there is a qualitative relation between an even-numbered cycle and its preceding odd numbered cycle. This property, which may be called the ‘tentative inverse G-O rule’, of even-numbered and preceding odd-numbered cycles, is violated by Cycle pairs (1, 2), (7, 8), and (17, 18). Excluding these cycle pairs, we find  $r = 0.77$  (significant on 98.48% confidence level), the slope is 3.3 times larger than the standard deviation, and  $\chi^2 = 36.5$  (highly significant on 5% level) of the best-linear fit between amplitudes of the remaining pairs ( $N = 9$ ) of odd- and the following even-numbered cycles (see Figure 3(b)). Overall, this relationship has a large uncertainty. In the case of small sunspot groups (maximum area smaller than 100 MSH), the inverse G-O rule found to be valid throughout Cycles 12–24 (Javaraiah, 2012). Using the relationships shown in Figure 3 and by knowing the epochs of violations of the G-O rule and the inverse G-O rule, it may be possible to predict the amplitude of a cycle from these rules (prediction for an even-numbered cycle is qualitative) except for the odd- and even-numbered cycles of the cycle pairs that violate the G-O rule and the inverse G-O rule. It may be noted here that the G-O rule and its violation has been known for a long time (see Komitov and Boney, 2001). The inverse G-O rule and its violations suggested above are speculative.

As can be seen in Figure 1, the epochs of the Cycle pairs (4, 5), (8, 9), and (22, 23) that violated the G-O rule are close to the epochs when the Sun’s orbital angular momentum is steeply decreased. Since it is possible to know the future epochs of the steep decrease in the Sun’s orbital angular momentum well in



**Figure 3.** Plots of amplitudes: (a) even-numbered cycles versus following odd-numbered cycles that satisfy the G-O rule and (b) odd-numbered cycles versus following even-numbered cycles that satisfy the tentative inverse G-O rule. The *continuous curve* represents the best linear fit and the *dashed curve* represents *rms deviation* ( $\sigma$ ). The values of the correlation coefficient ( $r$ ) and of the slope are also given, and near each data point, the corresponding cycle pair is shown.

advance, it seems possible to predict the violation of the G-O rule well in advance as well (Javaraiah, 2005). The orbital angular momentum of the Sun is steeply decreased during alternating 40-year and 139-year intervals. The next such a situation will arise in 2030 and after that, in 2169. Therefore, the next violation of the G-O rule is expected to occur in 2030, and after that, in 2169. The amplitude of Cycle 25 predicted to be lower than that of Cycle 24 (Javaraiah, 2015). Since small cycles usually have long periods, the upcoming Cycle 25 may also be a long cycle. We determined linear fit to the values of amplitudes and lengths of Cycles 1–23. The values of lengths (the time intervals between consecutive activity minima) were also taken from <http://www.ngdc.noaa.gov/>. ( $N = 23$ ,  $r = -0.35$  is small, also see Solanki *et al.*, 2002. The corresponding value of the 5% level significance is about -0.41 for 21 degrees of freedom.) Using the value 81.9 of the amplitude of Cycle 24 and the values 30–50 predicted for Cycle 25 (see Javaraiah, 2005 and below) in the best-fit linear equation, we obtain the values 11.4 and 11.7–11.9 for the lengths of Solar Cycles 24 and 25, respectively. Therefore, Cycle 24, which began in 2008.9, will probably end close to 2020, and Cycle 25 may end close to 2032 (these estimates have large uncertainties). These estimates suggest that the upcoming Cycle 25 will include the epoch 2030, when the next steep decrease in the orbital angular momentum of the Sun will take place. This cycle will be at the minimum between the current and the next Gleissberg cycles (see Figure 2).

The average level of activity is drastically different for different occasions of the steep decrease in the orbital angular momentum of the Sun (see Figure 1). Both the Maunder minimum (1645–1715) and Dalton minimum (1790–1830) included an epoch of the steep decrease in the orbital angular momentum of the Sun. However, in 1851 and 1990, when the orbital angular momentum of the Sun steeply decreased the level of activity was reasonably high. Between the dotted and dashed vertical lines in the intervals 1632–1672, 1811–1851, and 1990–2030, the activity decreased, increased, and decreased, respectively. The decreasing trend during 1990–2030 may lead to a considerably low activity around 2030, but it may be significantly higher than that in the Maunder minimum. A best linear fit to the values of amplitudes (164.5, 158.5, 120.8, 81.9, 30–50) and times of Cycles 21–25 (predicted values are used for Cycle 25) suggests 7–23 for the amplitude of Cycle 26, which is expected to begin close to the epoch 2030. Planetary configurations may cause modulations in the emerging magnetic flux (Javaraiah, 2005; Javaraiah and Bertello, 2016). It should be noted that no alignment of the major planets is exactly repeated (Jose, 1965).

Most of the solar system angular momentum is contributed by the orbital motions of the four giant planets, whose time-dependent spatial configurations are responsible for the irregular orbital motion of the Sun around the solar system barycenter. The orbital motion of Jupiter is the primary contribution (about 60%), and the Sun’s spin contributes about one to two percent to the total angular momentum of the solar system. (The net external torque on the Sun, due to orbital motions of the planets, is not zero, although it is small. So there could be some variation in the spin angular momentum of the Sun.) The solar rotation varies on many timescales (see Schröter, 1985; Snodgrass, 1992; Komm and Javaraiah, 2002; Antia, 2002; Javaraiah, Bertello, and Ulrich, 2005a,



2005b; Brajša, Ruždjak, and Wöhl, 2006; Javaraiah, 2013; Javaraiah and Bertello, 2016). Javaraiah (2005) found that significant (at the 99.5% confidence level) positive (before around 1945) and negative (after around 1945) correlations exist between the Sun's orbital torque and the solar equatorial rotation rate determined from sunspot group data during the period 1879–2004. Differential rotation and meridional flow are both important ingredients in flux-transport dynamo models (Dikpati and Gilman, 2006). Meridional circulation is primarily determined by the Coriolis forces from differential rotation, turbulent Reynolds stresses, and pressure and buoyancy forces (Rüdiger, 1989). An approximate linear relationship exists between the solar cycle variations in differential rotation and the meridional velocity of sunspot groups (Javaraiah and Ulrich, 2006). A high correlation (significant at the 99% confidence level) exists between the meridional flow and the Sun's orbital torque during Cycle 23 (personal communication from J. H. Shirley in 2017). (J.H. Shirley analyzed high-resolution JPL Horizons ephemeris data to determine the Sun's orbital torque and used the meridional flow speeds of small magnetic features on the Sun determined by Hathaway and Rightmire (2010) from observations of the *Michelson Doppler Imager* (MDI) on board the *Solar and Heliospheric Observatory* (SOHO) spacecraft spanning a period from May 1996 to June 2009. The values of the orbital torque and meridional flow speeds were averaged over time periods corresponding to one Carrington rotation, and the resulting 167-data-point time-series were cross correlated.) The variations in the Sun's spin-momentum may relate to variations in the solar differential rotation and therefore the solar dynamo (Zaqarashvili, 1997; Juckett, 2003; Wilson, Carter, and Waite, 2008). On the other hand, the Coriolis force due to solar rotation affects the rising magnetic flux through the convection zone to the photosphere due to magnetic buoyancy (Choudhuri and Gilman, 1987). Variations in solar rotation may be related to the variations in solar activity through the effect of Coriolis forces on the emerging solar magnetic flux.

The values of the Sun's orbit and spin angular momenta are  $\sim 5 \times 10^{47} \text{ g cm}^2 \text{ s}^{-1}$  (maximum) and  $\sim 2 \times 10^{48} \text{ g cm}^2 \text{ s}^{-1}$  (Antia, 2002), respectively. Javaraiah (2005) found that a good agreement exists between the amplitudes of the variations in the Sun's spin and the orbital angular momenta, particularly at the common epochs of the steep decreases in both the Sun's orbital angular momentum and the equatorial rotation rate determined from sunspot data. At these epochs the orbital angular momentum is low an order of magnitude approximately equal to its own magnitude (the magnitude varies from 0 to  $\sim 10^{47} \text{ g cm}^2 \text{ s}^{-1}$ ). For example, at the epoch 1990.97, it is found to be  $-2.1 \times 10^{47} \text{ g cm}^2 \text{ s}^{-1}$ , and there the amount of the drop in the equatorial rotation rate is found to be about 1% and the corresponding spin momentum is approximately equal to  $-1.1 \times 10^{47} \text{ g cm}^2 \text{ s}^{-1}$ . The epochs of violations of the G-O rule are close to the common epochs of the steep decrease in the orbital angular momentum of the Sun (when the rate of its change is slightly negative, *i.e.* when the orbital motion of the Sun is retrograde) and the equatorial rotation rate (see Figure 2 in Javaraiah, 2005). A correlation exists between the equatorial rotation rate and the latitude gradient of rotation (Balthasar, Vázquez, and Wöhl, 1986). Hence, a decrease in the equatorial rotation rate also implies a decrease in the differential rotation and leads to

a weak dynamo. A few weak cycles follow a large drop in the equatorial rotation rate (Javaraiah, 2003a). (Note that in the equatorial rotation rate a considerable drop occurred from Cycle 13 to Cycle 14 (see Balthasar, Vázquez, and Wöhl, 1986; Javaraiah, 2003a, 2003b; Javaraiah, Bertello, and Ulrich, 2005a), *i.e.* the Gleissberg cycle minimum around Cycle 14 also followed a considerable drop in equatorial rotation rate.) Ribes and Nesme-Ribes (1993) found that during the Maunder minimum, the equatorial rotation rate was lower by about 2%. Vaquero, Sánchez-Bajo, and Gallego (2002) found that it was lower by about 5%. (However, many studies show that rotation rate, equatorial rotation rate, and latitude gradient are higher during a solar cycle minimum than during maximum (*e.g.*, Balthasar, Vázquez, and Wöhl, 1986; Komm, Howard, and Harvey, 1993; Javaraiah and Komm, 1999; Javaraiah, 2003b; Brajša, Ruždjak, and Wöhl, 2006). The large drops in the long-term rotation variation may originate at deeper layers of the Sun, while the high rotation rate during a solar cycle minimum may represent the rotation rate of small magnetic regions at subsurface and surface layers, see Javaraiah, 2013; Javaraiah and Bertello, 2016 and references therein).

The violation of G-O rule is followed by at least one relatively weak cycle (see Figure 1). Since Cycles 6, 10, and 24 are weaker than Cycles 5, 9, and 23, respectively, Cycle 26 will be a weak cycle whose strength may be close to that of Cycle 25 (note that as argued above the Cycle pair (24, 25) is expected to violate the G-O rule, and also note the prediction of Javaraiah (2015)). Komitov and Boney (2001) have found that violation of the G-O rule cannot be a random phenomena, but occurs under special conditions, the main factor being the very high maximum of the even-numbered cycle, for example, Cycles 4, 8, and 22. Since the maximum of Cycle 24 is not very high, it does not support the notion that the Cycle pair (24, 25) will violate the G-O rule as predicted by Javaraiah (2015).

The basic concept of the prediction of Javaraiah (2015) is the existence of a high correlation ( $r$  is 0.96–0.97) between the sum of the areas of sunspot groups in the  $0^\circ - 10^\circ$  interval of the southern hemisphere during the time interval of 1.0 year to 1.75 year just after the time of the maximum of a cycle and the amplitude of its immediately following cycle (Javaraiah, 2007). A statistically high significant linear relationship exists between these quantities (*cf.* Equation (2) in Javaraiah, 2007; Equation (3) in Javaraiah, 2015). In the case of Cycle 24, the second peak of  $R_Z$  is larger than the first peak, which is unusual. The first peak with a value of 66.9 occurred in 2012.17, while the large second peak with a value of 81.9 is occurred in 2014.3. Javaraiah (2015) analyzed the combined Greenwich and *Solar Optical Observing Network* (SOON) sunspot group daily data during the period 1874–2013 (downloaded from the website <http://solarcience.msfc.nasa.gov/greenwich.shtml>) and determined the sums of the areas of sunspot groups in the different  $10^\circ$  latitude intervals of the northern and southern hemispheres during different time intervals. By using the value 18.86 of the sum (divided by 1000) of the areas of sunspot groups in  $0^\circ - 10^\circ$  latitude interval of the southern hemisphere during the interval 2013.17–2013.92, which is one year away from the epoch of the first peak of Cycle 24,

Javaraiah (2015) predicted  $50 \pm 10$  for the amplitude of the upcoming solar Cycle 25. Here we analyzed the extended Greenwich-SOON data (extended with the SOON sunspot group daily data for the period 2014–2016) and determined the sum of the areas of sunspot groups in the  $0^\circ - 10^\circ$  latitude interval of the southern hemisphere during the interval 2015.30–2016.05 which is one year away from the epoch of the large second peak of Cycle 24. This sum is found to be only 5.18. Hence, we obtain a much lower value  $29.9 \pm 10$  for the amplitude of Cycle 25. Here we have also analyzed the combined updated Greenwich and Debarcen Observatory sunspot group daily data (DPD) during the period 1874–2016, which were downloaded from the website <http://fenyi.solarobs.unideb.hu/pub/DPD/> (for details see Győri, Baranyi, and Ludmány, 2010). We find a relation that is same as the Equation (3) in Javaraiah (2015). The sums of the areas of sunspot groups in the  $0^\circ - 10^\circ$  latitude interval of the southern hemisphere during the time intervals 2013.17–2013.92 and 2015.30–2016.05 are found to be 28.19 and 11.0, respectively. Using these values, we derive the values  $65 \pm 11$  and  $39 \pm 11$  for the amplitude of Cycle 25. As we have pointed out earlier (Javaraiah, 2007, 2008, 2015), the interval of 1.0–1.75 year after a cycle maximum epoch is close to (or includes) the epoch of the reversal of the Sun’s polar magnetic field (see Table 1 in Makarov, Tlatov, and Sivaraman, 2003). The magnetic field in the southern hemisphere reversed its polarity in mid-2013 (Mordvinov *et al.*, 2016), *i.e.* within the intervals 2013.17–2013.92, which is one year away from the epoch of the first peak of Cycle 24 (there may be some transport of magnetic flux across the solar equator, the combined effect of the Sun’s rotation and the inclination of the Sun’s equator may play a role). The values, which were obtained for the amplitude of Cycle 25 using the sums of the areas of the sunspot groups in this interval, seem to be fine. However, the low values (30–40) predicted using the sums of the areas of sunspot groups in the interval 2015.30–2016.05, which is one year away from the epoch of the large second peak of Cycle 24, are more relevant to the basic concept described above. These values imply that Cycle 25 will be a much weaker cycle.

The values  $74 \pm 10$  (Javaraiah, 2005) and  $87 \pm 7$  (Javaraiah, 2008) which were predicted for the amplitude of Cycle 24 are close to the real value, *i.e.* these values match (within their uncertainty limits) the value 81.9 of the maximum amplitude of Cycle 24 (all the other predictions in the two articles cited last have lower uncertainties, and all indicated that Cycle 24 is weaker than Cycle 23). Hence, mostly the amplitude of Cycle 25 will be lower than that of Cycle 24, as predicted in Javaraiah (2015) and here. On the other hand, although we here predict much lower values for the amplitude of Cycle 25 by analyzing two data sets, assuming that the prediction fails, *i.e.* the Cycle pair (24, 25) will not violate the G-O rule, hence, the property that violation of G-O rule is followed by at least one weak cycle will not be applicable for Cycle 26. However, Cycle 26 is still expected to be weaker than Cycle 25. This is because an epoch of the steep decrease in the orbital angular momentum of the Sun seems to be followed by a relatively weak cycle, and Cycle 26 will follow the steep decrease in the orbital angular momentum of the Sun at 2030. This is also consistent with the tentative inverse G-O rule, as well as with the decreasing trend of the activity from Cycle 22, *i.e.*, from the last dotted vertical line in Figure 1, as suggested

above. Even if the Cycle pair (25, 26) will not satisfy the inverse G-O rule, the amplitude of Cycle 26 will still not be much higher (see the trend in the extrapolated portion of the cosine curve in Figure 2(b)).

The violation of the tentative inverse rule also seems to occur in somewhat regular intervals. As we described above, the Cycle pairs (1, 2), (7, 8), and (17, 18) violated this tentative rule. The first of these pairs is in the fourth quarter of the 139-year interval of the orbital angular momentum of the Sun, which comprises part of the Maunder minimum, the second one is in the 40-year interval, which comprises part of the Dalton minimum, and the third one is in the fourth quarter of the 139-year interval, which comprises the modern minimum (1890–1939). This sequence shows that the next one should take place in the 40-year interval that comprises the current deep minimum. However, the difference between the epochs of the first two pairs that violated the tentative inverse G-O rule is four cycles (3–6), while the differences between the second and third pairs is eight cycles (9–16). Already more than five cycles elapsed since the last Cycle pair (17, 18) that violated the tentative inverse G-O rule, and as inferred above, Cycle pair (25, 26) is expected to obey the tentative inverse G-O rule. Thus, if we consider the difference of eight cycles, the next cycle pair that will violate the tentative inverse G-O rule will be cycle pair (27, 28). Hence, Cycle 27 will be weaker than Cycle 28. In addition, Cycles 1, 7, and 17 are stronger than their respective preceding even-numbered cycles. Hence, we can also infer that Cycle 27 will be stronger than Cycle 26. As described above, the next epoch of the violation of G-O rule is much farther away, hence Cycle 29 will be stronger than Cycle 28. Even when we consider the difference of four cycles, after Cycle pair (27, 28), the next cycle pair that is expected to violate the tentative inverse G-O rule may be cycle pair (33, 34). Hence, Cycle 29 is also expected to be stronger than Cycle 30. Cycle 29 will probably be a substantially strong cycle, and its maximum may represent the upcoming Gleissberg maximum. However, we cannot rule out the possibility that the maximum of Cycle 31 represents the upcoming Gleissberg maximum, as we can estimate from the curve shown in Figure 2(b). Overall, the inferences above are consistent with the pattern of the extrapolated portion of the curve in Figure 2(b), *i.e.* the values of the amplitudes of Solar Cycles 25–30 are expected to be close to the corresponding extrapolated values and satisfy the aforesaid inferences and predictions (listed in the next section) on the relative amplitudes of these solar cycles.

### 3. Conclusions and discussion

The solar activity varies on many timescales. The study of variations in solar activity is important for understanding the underlying mechanism of solar activity and for predicting the level of activity in view of the activity impact on space weather and global climate. Here we have used the amplitudes of Sunspot Cycles 1–24 to predict relative amplitudes of the solar cycles during the rising phase of the upcoming Gleissberg cycle. We fitted a cosine function to the amplitudes and times of solar cycles, after subtracting a linear fit of the amplitudes. The best cosine fit shows overall properties (periods, maxima, minima, *etc.*)

of Gleissberg cycles, but with large uncertainties. We obtained a pattern of the rising phase of the upcoming Gleissberg cycle, but there is considerable ambiguity. Using the epochs of violations of the Gnevyshev-Ohl rule (G-O rule) and the ‘tentative inverse G-O rule’ of solar cycles during the period 1610–2015, and also using the epochs where the orbital angular momentum of the Sun is steeply decreased during the period 1600–2099, we inferred that i) Solar Cycle 25 will be weaker than Cycle 24, ii) Cycles 25 and 26 will have almost same strength, and the epochs of these cycles will be at the minimum between the current and upcoming Gleissberg cycles, iii) Cycle 27 is expected to be stronger than Cycle 26 and weaker than Cycle 28, iv) Cycle 29 is expected to be stronger than both Cycles 28 and 30, and v) the maximum of Cycle 29 is expected to represent the next Gleissberg maximum. Our analysis also suggests a much lower value (30–40) for the maximum amplitude of the upcoming Cycle 25.

It should be noted that the physical connection between the solar system dynamics and solar activity is not clear yet (Zaqarashvili, 1997; Juckett, 2003; Gokhale, 2010; Wolff and Patrone, 2010; Abreu *et al.*, 2012; Cionco and Compagnucci, 2012; Wilson, Carter, and Waite, 2008; Wilson, 2013; Chowdhury *et al.*, 2016; Stefani *et al.*, 2016). However, the aim of the present analysis is not to establish this connection. The relatively short-term predictions of the solar activity that were made based on the hypothesis of a role of solar system dynamics in the mechanism of solar activity have failed (Meeus, 1991; Li, Yun, and Gu, 2001). Charvátova (2008) used the motion of the Sun about the barycenter to infer that Cycles 24–26 will be repeat Cycles 11–13, and they predicted 140 (also 100), 65 and 85 for the amplitude of Cycles 24, 25 and 26, respectively. Obviously, the prediction for Solar Cycle 24 failed, but the trend over Cycles 24–26 is consistent with our prediction that both Solar Cycles 25 and 26 will be weaker than Cycle 24 (conclusions (i) and (ii) above). However, only the hypothesis of a role of solar system dynamics in the mechanism of solar activity is not a main factor in the present analysis. In fact, only one necessary piece of information, namely the epochs of the steep decreases in the orbital angular momentum of the Sun, was used for our predictions above (conclusions (i)–(v) above). According to these epochs, the violation of the G-O rule is much farther way after the violation in 2030 (Javaraiah, 2005), *i.e.* after the G-O rule has been violated by Cycle pair (24, 25). Javaraiah (2015) predicted a violation of the G-O rule Cycle pair (24, 25), and here the results from the analysis of two extended datasets strongly support it. We have extensively extracted the necessary information from the trends in the yearly and the cycle-to-cycle variations in  $R_Z$  (*i.e.*, mainly used the epochs of the violations of the G-O and inverse G-O rules).

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